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Optimization of Low-Altitude Global Communication Constellations

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The problem of optimizing the orbital structure of multisatellite networks designed to transfer information globally is considered. A new criterion, the number of independent intersatellite cross-link paths between any pair of satellites in the constellation, is proposed. A problem of orbital pattern structure optimization is formulated. A numerical solution algorithm is described. Results of the problem solution in the set of kinematically regular networks are given. An estimation of the maximum possible number of independent intersatellite cross-link paths is proposed. Comparisons of calculation results and estimation show high performance of kinematically regular constellations.

Nomenclature

C_n, C_m = symbols for n- and m-order cyclic groups
c_ψ = a rotation through the angle ψ around tangent to equatorial plane axis intersecting the Earth's center
E = unity group symbol
e = unity transformation of satellite phase state
e' = transformation of satellite phase state changing only direction of satellite motion
G = graph of intersatellite cross links
G_{jk} = subgraph of G where every node of a class K_{jk}^{l+1} is linked at least by an edge with some node of the class K_{jk}^l
h_{atm} = height of radio-opaque atmosphere
i = orbits' inclination
K_{jk}^l = class of division of graph G
M = number of independent cross-link paths in a given network
M_{jk} = number of independent cross-link paths between satellites j and k
M_{jk}^a = current actual number of independent cross-link paths between satellites j and k during the algorithm procedure
M_{jk}^p = current number of cross-link paths between satellites j and k that can possibly turn into independent ones later during the algorithm procedure
$M_{\max}$ = maximum number of independent cross-link paths for given N
m, n = orders of specific cyclic groups defining network symmetry group
N = number of satellites in the network
q = number of graph G division classes for selected pair j,k
R_E = mean radius of the Earth, 6371.2 km
r = radius of network satellites' orbits
S = the set of all possible states of constellations providing continuous whole Earth coverage
s_j = phase state of a satellite
s = phase state of a network
t_{jk}^d = time instants when satellites j and k are at line-of-sight limit
U = group consisting of all possible displacements u_τ of network satellites along their orbits
u_τ = changing of satellite's phase by τ
V = second-order group consisting of transformations e and e'
W = group consisting of all turns around all possible axes crossing the Earth's center
x_j, y_j, z_j = Cartesian coordinates of the satellite j in geocentric coordinate system (OZ is tangent to equatorial plane, OX coincides with reference direction of nodal ascension): $x_j = r[\cos\Omega_j \cos(\vartheta_j + \theta) - \sin\Omega_j \sin(\vartheta_j + \theta)\cos i]$; $y_j = r[\sin\Omega_j \cos(\vartheta_j + \theta) + \cos\Omega_j \sin(\vartheta_j + \theta)\cos i]$; $z_j = r\sin(\vartheta_j + \theta)\sin i$
α = geocentric angular radius of a satellite coverage zone
$\alpha_{\min}$ = minimum value of α required for a constellation to cover the Earth's surface continuously
ϵ = elevation angle
θ = time measured in units of satellite phase
θ_{jk}^d = the values t_{jk}^d expressed in units of satellite phase
κ = integer defining automorphism of m- and n-order cyclic group (two networks possessing group symbols differing but with κ element differ only with phase spacing in adjacent orbit planes)
μ = Earth's gravitational constant, 398,600 km^3/s^2

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ξ_{jk} = supplementary variable for computation of moments t_{jk}^d, $\cos(\vartheta_j - \vartheta_k)$

$$+\frac{2\cos\varphi_{\max} + 2\sin(\vartheta_j - \vartheta_k)\sin(\Omega_j - \Omega_k)\cos i - 2\cos(\vartheta_j - \vartheta_k)\cos(\Omega_j - \Omega_k)}{\sin^2 i\,[\cos(\Omega_j - \Omega_k) - 1]}$$

ϑ_j = initial (at moment $t = 0$) phase of the satellite j, i.e., measured in the orbit plane geocentric angle between the satellite and its ascending node

Σ_1, Σ_2 = areas upon unity sphere surface disposed outside caps of geocentric radii $\varphi_{\max}$ and $\varphi_{\max} + \alpha$, respectively: $\Sigma_1 = 2\pi(1 + \cos\varphi_{\max})$; $\Sigma_2 = 2\pi[1 + \cos(\varphi_{\max} + \alpha)]$

$\varphi_{\max}$ = geocentric angle between satellites being at line-of-sight limit, $2\cos^{-1}[(R_E + h_{\text{atm}})/r]$

Ω_j = initial right ascension of ascending node of the satellite j

Subscripts

j,k = indices of satellites in the network run from 1 to N

Superscripts

d = index of moments in the set t_{jk}^d runs from 1 to 4

l = index of graph G division class runs from 1 to q

Introduction

THE existing communication satellite networks use as a rule high orbits—geostationary or highly elliptical. Communication between remote ground stations is accomplished through the chain Earth-satellite-Earth. The disadvantages of such a network structure are the relatively slow transfer of signals when global communication is needed and the relatively high capacities of satellite transmitters. These disadvantages might be considerably diminished by using low-altitude satellite networks (with orbit heights below 5000 km) in which information is transferred through intersatellite communication links. This technique is used in multiple satellite networks.[1] A specific problem of intersatellite link set connectivity occurs for the networks owing to their low altitude. This problem is discussed in Ref. 2.

This paper addresses the problem of the design of a network orbital pattern with a maximum number of independent intersatellite cross-link paths.

The solution of the problem is sought in a class of constellations known as kinematically regular. The constellations were proposed by Mozhaev[3] for the solution of the continuous coverage problem. They were obtained by development of symmetry theory for evenly rotating satellites. For more details, the reader should turn to Refs. 4 and 5. In the West, some of the kinematically regular constellations are better known as "delta patterns."[6,7] The name was given by Walker, who came to the constellations independently, using geometrical considerations. Since then, a number of studies appeared that examined the application of the patterns for different purposes: multiple continuous Earth coverage,[8,9] Navstar/Global Positioning System (GPS),[10] and minimization of revisit time of regions on the Earth surface.[11] The broadening of the kinematical regularity conception to the realm of elliptic orbits leads to patterns that provide continuous whole Earth coverage by only a four-satellite network.[12]

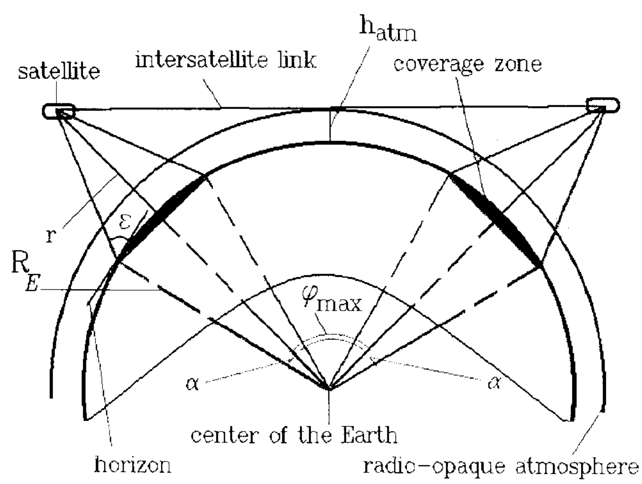


Fig. 1 Geometry of information transfer.

Thus, kinematically regular constellations showed themselves to be very popular for different applications. Another argument in their favor is the following: there is no changing of one satellite initial position in the network with respect to the rest that can improve the performance of the network (the change of one satellite position is simply insufficient owing to the regularity of the network). The condition might be considered necessary for the network's optimality.

Problem Formulation

Consider a communication network consisting of N satellites orbiting the Earth. It is assumed to be a sphere of radius R_E. All satellites have the same radius r and inclination i. With insignificant effects on the solution of the problem concerned, it may be presumed that satellites rotate evenly with the period

$$T = 2\pi\sqrt{r^3/\mu}$$

Under this presumption, the satellite motion is completely defined by initial values of parameters i, Ω_j, and ϑ_j. The triad composes the phase state of a satellite: $s_j = (i, \Omega_j, \vartheta_j)$. Phase state s of a network is the amalgam state of its satellites: $s = (s_1, s_2, \ldots, s_N)$.

Every satellite is able to communicate with objects on the Earth's surface disposed within its coverage zone—a subsatellite point centered small circle having geocentric radius α. It can also communicate with other satellites directly visible (Fig. 1). Direct visibility between satellites is limited by height h_{atm}. If communication between two satellites is possible, the pair could be joined by the straight line that is distant from the Earth's center no less than $R_E + h_{atm}$. Hereafter, the line is called an intersatellite link or link for short. A message from one satellite to another is transferred through some communication route consisting of links and satellites joined with them. Two routes linking the same pair of satellites are called independent cross-link paths if they do not include common satellites except for source and recipient.

The networks under consideration are to provide global communication, i.e., two ground stations may exchange information through the network wherever they are disposed on the Earth's surface. Thus continuous global coverage should be guaranteed as a necessary condition for the network performance. The networks considered in the following are assumed to fulfill the condition.

The main task of the network consists of the reliable transfer of information from one ground station to another through the set of intersatellite links, provided that some of its satellites cannot perform the transmission (are busy at the moment or failed). It is reasonable to consider M_{jk} as a quantitative measure of reliability for the given pair of satellites j and k at the given moment t. It is obvious that $M_{jk} = M_{jk}(s,t)$. Then the information transfer reliability measure for a network with initial state s is

$$M(s,t) = \min_{j \neq k} M_{jk}(s,t) \tag{1}$$

During rotation period T a value of $M_{jk}(s,t)$ changes, and, thus, it is natural to take the value

$$M(s) = \min_{0 \leq t < T} M(s,t) \tag{2}$$

as a criterion of network performance reliability: the last increases with enhancement of value (2).

Thus, the problem could be formulated in the following terms: in a set of continuous coverage constellations, a constellation should be found with initial state minimizing value (2). In other words, it is necessary to find an extremum

$$M_{\max} = \max_{s \subset S}\{\min_{0 \leq t < T}[\min_{j \neq k} M_{jk}(s,t)]\} \tag{3}$$

Criterion Calculation Algorithm

The numerical solution of the problem implies computation of the criterion $M(s)$. This has a simple enough interpretation in terms of graph theory. Indeed, at a given instant t, a set of intersatellite links may be plotted in the form of graph G with edges representing links and nodes representing satellites. The structure of G depends totally on initial state s of the network and instant t, i.e., $G = G(s,t)$. Considering all possible cuts of graph $G(s,t)$ during time interval $[0,T)$ select a cut having a minimum number of edges. This number is nothing but $M(s)$.

The complete set-of-cuts analysis requires significant expenditures of computer resources even for graphs with a small number of nodes. An algorithm proposed in the following allows the calculation of a value of $M(s)$ with acceptable expenditures. In the basis of the algorithm lies the computation of the number $M_{jk}(s,t)$.

The following procedure for M_{jk} computation can be proposed. Divide a set of graph G nodes into q classes $K_{jk}^0, K_{jk}^1, \ldots, K_{jk}^q$ by the following rule (Fig. 2): each class K_{jk}^0 and K_{jk}^q comprises one node j and k, respectively; the class K_{jk}^1 comprises all of the nodes contiguous to j except, perhaps, k, each class K_{jk}^l ($l = 2,3,\ldots,q-1$) comprises all of the nodes contiguous to nodes of class number $(l-1)$ with exception of, perhaps, k and any other included into classes $K_{jk}^{l-1}, K_{jk}^{l-2}, \ldots, K_{jk}^1$ (a node should never belong to two different classes). The number q is determined by the structure of $G = G(s,t)$ and the selection of pair j,k (at Fig. 2 $q = 4$).

Obviously, the number of independent cross-link paths cannot exceed a number of edges contiguous to node j or k.

Assume at first that every node of class K_{jk}^l ($l = 1,2, \ldots, q-1$) is linked by at least one edge with some node of the class K_{jk}^{l+1} (subgraph $G_{13\,11}$ at Fig. 2). Under this assumption, the algorithm takes the following form.

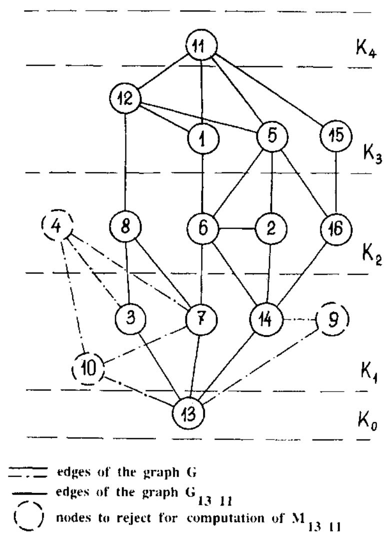


Fig. 2 Graph G: division into classes and isolation of information transferring subgraph.

Compute, subsequently, numbers of edges linking classes K_{jk}^0 and K_{jk}^q, K_{jk}^0 and K_{jk}^1, K_{jk}^1 and K_{jk}^q, K_{jk}^1 and K_{jk}^2, and so forth.

The number of edges linking classes K_{jk}^0 and K_{jk}^q determines the number M_{jk}^a of the paths already found.

Consider classes K_{jk}^0 and K_{jk}^1. A number of edges linking classes K_{jk}^0 and K_{jk}^1 determines the number M_{jk}^p, which characterizes remnant independent path ability of the node j. It means that even if the number of edges linking classes K_{jk}^1 and K_{jk}^q exceeds the current value of M_{jk}^q the full number of independent paths will never be more than $M_{jk}^a + M_{jk}^p$. Now turn to K_{jk}^1 and K_{jk}^q. If a number of edges linking the classes is equal or exceeds M_{jk}^p, then the procedure stops and $M_{jk} = M_{jk}^a + M_{jk}^p$, otherwise M_{jk}^a should be increased and M_{jk}^p should be decreased by the number of edges linking the classes.

Then classes K_{jk}^1 and K_{jk}^2 are to be analyzed. If the number of edges linking the classes is less than the current value of M_{jk}^p, then M_{jk}^p should be corrected (reduced to the number). In other words, a number of independent paths that may actualize in consequent classes is the least of two values: newly obtained M_{jk}^p and a number of edges linking classes K_{jk}^1 and K_{jk}^2.

The computational procedure for numbers M_{jk}^a and M_{jk}^p described earlier should be continued for pairs K_{jk}^2 and K_{jk}^q, K_{jk}^2 and K_{jk}^3, K_{jk}^3 and K_{jk}^q, K_{jk}^3 and K_{jk}^4, and so forth. The procedure stops when M_{jk}^p reaches 0 or all classes are looked through. A value of M_{jk}^a stored at the end determines the number M_{jk} sought after.

The algorithm just described is valid only under the assumption previously adopted, otherwise an excessive value of M_{jk}^p may appear during calculations distorting results.

To avoid the nuisance, we must previously extract out of G a subgraph G_{jk} responsible for information transfer from node j to node k (subscripts are applied to show that structure of the subgraph depends on selection of j and k). To find G_{jk}, it is necessary after subdivision of nodes into classes to search consecutive classes $K_{jk}^{q-1}, K_{jk}^{q-2}, \ldots, K_{jk}^1$ and reject all of the nodes lacking contiguity to nodes from classes $K_{jk}^q, K_{jk}^{q-1}, \ldots, K_{jk}^2$, respectively, as well as edges contiguous to these defaulter nodes (Fig. 2). To find $M_{jk}(s,t)$ the algorithm just described should be applied to graph G_{jk}.

For computation of the extremum (1), all pairs jk ($j \neq k$) must be analyzed and, thus, all of the graphs G_{jk} are to be built.

Now let us consider the extremum (2) computation procedure.

It is not as time consuming as it looks because within the interval $[0,T)$, the structure of graph G does not change continuously. Indeed, such change is possible only when some satellite appears in or disappears off sight of some other satellite. Thus, to calculate minimum (2), it is enough to compute minimum (1) for the finite set of time instants. The instants are those between two subsequent moments when two satellites are in line-of-sight limit.

The critical moments can be obtained by solving

$$(x_j x_k + y_j y_k + z_j z_k)/r^2 = \cos \varphi_{\max} \qquad (j \neq k)$$

After substitutions and reductions, the following equation for time instants in which angular distance between satellites j and k equals $\varphi_{\max}$ can be obtained:

$$\cos(2\theta + \vartheta_j + \vartheta_k) = \xi_{jk}$$

For every pair jk in the interval $[0,2\pi)$, the equation either has four roots or no roots at all. If the roots do exist ($\xi_{jk} \leq 1$), they are

$$\theta_{jk}^1 \equiv \tfrac{1}{2}(\cos^{-1}\xi_{jk} - \vartheta_j - \vartheta_k) \bmod 2\pi$$
$$\theta_{jk}^2 \equiv \tfrac{1}{2}(2\pi - \cos^{-1}\xi_{jk} - \vartheta_j - \vartheta_k) \bmod 2\pi$$
$$\theta_{jk}^3 \equiv (\pi + \theta_{jk}^1) \bmod 2\pi$$
$$\theta_{jk}^4 \equiv (\pi + \theta_{jk}^2) \bmod 2\pi$$

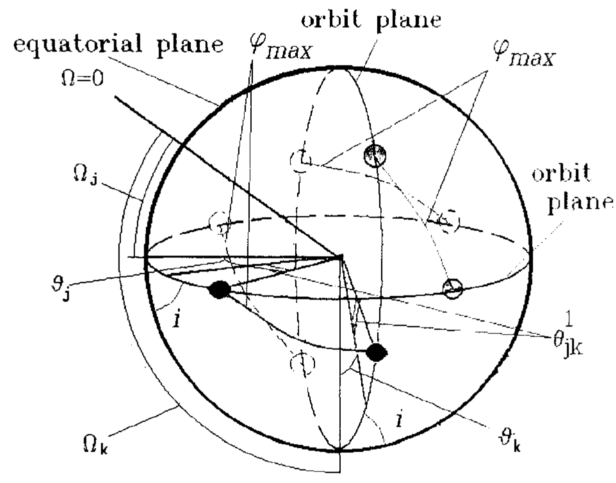


Fig. 3 Critical moments of links' installment and disinstallment.

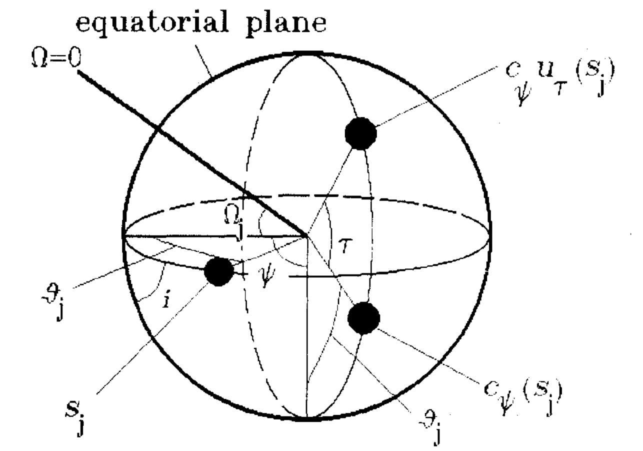


Fig. 4 Effect of the group $\{W,U,V\}$ transformations on satellite position.

Absence of the roots means that satellites either have a link during the whole period T or can never install a link (e.g., those sharing the same orbit and parted by phases gap π).

Figure 3 represents the mutual position of a satellite pair at critical moments. The instants of time when graph G changes its structure during rotation period are

$$t^d_{jk} = \theta^d_{jk}\left(\frac{T}{2\pi}\right) \tag{4}$$

$$(j = 1,2,\dots,N;\ k = 1,2,\dots,N;\ j \neq k;\ d = 1,\dots,4)$$

At every moment out of the set (4), a link between a pair of satellites appears or disappears. A number of independent cross-link paths can drop only as a result of link disappearance. Hence, to find the minimum [Eq. (2)] we must extract from the set (4) all of the moments of link disappearance (they total half of the set), construct graph G at every moment found, and apply the algorithm described earlier to every graph built.

Description of Selected Continuous Coverage Set

It seems impossible at least now to find a strict value of s providing the maximum [Eq. (3)] because of substantial computational expenditures. Moreover, knowledge of the strict value is not necessary because the solution of the problem is multivalued. Indeed, the criterion M_{max} is highly symmetrical: it does not change when a network like a solid structure is turned around any axis intersecting the Earth's center or reflected in the point; it also does not change if all satellites of the network undergo simultaneous and equal changing of their phase angles or reverse simultaneously the direction of their orbital motions. In other words, M_{max} is invariant with respect to transformations of the group $\{W,U,V\}$ (generated by groups W,U,V), which is called the symmetry group of criterion M_{max}.

The symmetry group of criterion (3) coincides with that of continuous coverage criterion α_{min} analyzed in Refs. 3–5. Following Ref. 3, optimal constellations are sought only within the set of those invariant with respect to some finite subgroups of $\{W,U,V\}$. In this study, only subgroups having specific cyclic components of the group W with the axis tangent to the equator plane are considered. Generated by the groups, regular networks, also known as "delta patterns," were studied in Refs. 6–11 and showed high performance. It must be noted, however, that values of radii α_{min} for constellations consisting of more than 24 satellites apparently were never published previously.

The initial state of any kinematically regular network is totally determined by its symmetry group and the initial state of one (any) of its satellites. It was shown in Ref. 4 that the criterion value for networks possessing groups of the selected type is independent from the initial position of the satellite (let it be satellite 1 for certainty). The first satellite may always have initial parameters $\Omega_1 = 0$ and $\vartheta_1 = 0$ (parameter i is not mentioned because it is not affected by group transformations). Classification of networks is reduced to the classification of groups described in Ref. 4.

In this study, the group notification adopted in Ref. 4 is used, which requires a short reminder. Group structure is coded in its symbol. For any group of the selected type, the symbol should have the general form $NC_nC_m\kappa$ with the following exceptions: symbol C_n (respectively, C_m) is replaced by symbol E when $n = 1$ ($m = 1$), and symbol κ is omitted when $n/m = 1, 2,4,6$. Integers n,m and κ follow certain rules: N/n and n/m are integers, κ and n/m are coprimes, $\kappa < n/2m$.

To construct all groups of given order N, calculate all factors n of N, then for every n, find all factors m; for every pair (n,m), calculate all numbers κ following the previously described constraints. Using the numbers obtained, construct all of the symbols possible in accordance with rules mentioned earlier. For instance, if $N = 6$, then $n = 1,2,3,6$; for $n = 1$ $m = 1$; for $n = 2$ $m = 1,2$; for $n = 3$ $m = 1,3$; for $n = 6$ $m = 1,2,3,6$; since n/m in all of the cases belongs to the set $(1,2,3,6)$, κ is omitted. The total set of symbols is as follows: $6EE$, $6C_2E$, $6C_2C_2$, $6C_3E$, $6C_3C_3$, $6C_6E$, $6C_6C_2$, $6C_6C_3$, $6C_6C_6$.

The structure of a group is described in the following in terms of generating elements, i.e., the group consists of all possible mutual products of the elements raised in all possible powers.

Consider two transformations of satellite phase state c_ψ and u_τ. Their effects on s_j are given by following equations:

$$c_\psi(i,\Omega_j,\vartheta_j) = (i,\Omega_j + \psi,\vartheta_j)$$
$$u_\tau(i,\Omega_j,\vartheta_j) = (i,\Omega_j,\vartheta_j + \tau) \tag{5}$$

Figure 4 represents satellites in states $s_j, c_\psi(s_j)$, and $c_\psi u_\tau(s_j)$. Writing $c_\psi u_\tau(s_j)$ means that at first s_j is transformed by u_τ and later by c_ψ; $c^\kappa_\psi(s_j)$ means that (s_j) should be transformed κ times by $c_\psi[c^0_\psi(s_j) = e(s_j) = s_j]$.

Table 1 Initial state parameters of regular network possessing symmetry group $6C_3C_3$

	Satellite number j					
	1	2	3	4	5	6
Ω_j	0	$2\pi/3$	$4\pi/3$	0	$2\pi/3$	$4\pi/3$
ϑ_j	0	π	0	π	0	π

Generating elements for the group given by the symbol $NC_nC_m\kappa$ are

$$c^{\kappa}_{2\pi/n}u_{2\pi m/N}, \qquad c_{2\pi/m}$$

For example, a group $6C_3C_3$ has the following structure

$$6C_3C_3 = \bigcup_{j=0}^{5} (c_{2\pi/3}\,u_{\pi})^j$$

(Subgroup generated by element $c_{2\pi/3}$ is enclosed.)

The state s of a network determined by given group H can be obtained from the state $s_1 = (i,0,0)$ subject to group's transformations according to Eqs. 5. The transformations are those mutually unidentical from the set

$$\left[\bigcup_{j=0}^{nN/m-1} (c^{\kappa}_{2\pi/n}u_{2\pi m/N})^j\right] \bigcup \left[\bigcup_{j=0}^{m-1} c^{j}_{2\pi/m}\right]$$

(There are only N mutually unidentical transformations in the set and they make up the group H.) For the mentioned group $6C_3C_3$, parameters Ω_j and ϑ_j of the initial state are represented in Table 1. In notation[9] better known to Western readers, the descriptor of the constellation is 6/3/3.

To find extremum (3) in the selected subset of kinematically regular networks, all their symmetry groups should be constructed as described earlier; by using them, the initial states should be built. Then for every structure obtained, the only free parameter i is to be optimized by criterion (2). The range of inclination to vary can be derived by the solution of the whole Earth coverage problem in the selected constellation class.

For the purpose of every constellation of the class, a function $\alpha_{\min}(i)$ should be computed by the algorithm described in Ref. 4. The obtained function file or catalogue can be very helpful because various constellation design problems require continuous whole Earth coverage as a precondition. Inclination ranges represented in Tables 2 and 3 were obtained from such catalogues.

Numerical Results: Estimation of Performance Limits

Though α is a convenient parameter for constellation performance analysis, more frequently dimensions of coverage zone are given by minimum value of ϵ, at which the satellite is still visible from the point on the Earth surface (Fig. 1). The relation between the two is

$$\alpha = \cos^{-1}[(R_E/r)\cos\epsilon] - \epsilon \tag{6}$$

In this paper, calculations were performed for $\epsilon = 5^0$. The value is close to the practical limit obtainable for a sea-based network user (for a land-based user, the value is usually higher). Another value assumed is radio-opaque atmosphere height $h_{\text{atm}} = 100$ km.

Two numerical researches were performed. During the first one, numbers of independent cross-link paths for 2000-km-high networks ($r = 8371.2$ km) consisting of 25, 27, 30, and 35 satellites were calculated. In the second one, calculations of the paths number for 25 satellites networks at height 2000, 2500, 3000, and 4000 km were performed. Substitution of respective values in Eq. (6) gives the following limit values of $\alpha_{\min}$ for each of selected heights: 35.69^0, 39.32^0, 42.37^0, and 47.26^0. Comparison of these values with functions $\alpha_{\min}(i)$ in the catalogues mentioned earlier produces ranges of inclination to vary during the cross-link paths calculation procedure, i.e., defines the set S.

Results of the first research are presented in Table 2. It contains symmetry groups of all the networks submitting coverage requirements at height 2000 km and inclination ranges in which network coverage zone radii exceed 35.69^0. In column $M^{\max}$, the peak number of independent cross-link paths in the given inclination range is printed.

Table 2 Numbers of independent cross-link paths in regular networks at height 2000 km

Group's symbol	Inclination range, deg	$M^{\max}$	Group's symbol	Inclination range, deg	$M^{\max}$
$25C_{25}E6$	71–81	8	$30C_{15}C_5$	58–94	9
$25C_{25}E7$	113–121	8	$30C_{30}C_24$	58–94	10
$25C_{25}E9$	118–123	8	$30C_{30}C_27$	58–94	9
$27C_3E$	87–95	7	$30C_{30}C_27$	114–123	10
$27C_3C_3$	86–93	8	$35C_5E1$	56–121	12
$27C_9E2$	56–68	8	$35C_5E2$	56–124	12
$27C_{27}E5$	57–70	9	$35C_5C_5$	56–121	12
$27C_{27}E7$	90–116	7	$35C_7E_1$	57–71	12
$27C_{27}E10$	57–65	9	$35C_{35}E4$	60	12
$27C_{27}C_3$	117–120	9	$35C_{35}E6$	57–118	12
$30C_3E$	81–100	11	$35C_{35}E8$	56–76	12
$30C_3C_3$	81–99	11	$35C_{35}E8$	93–123	11
$30C_5E1$	80–102	10	$35C_{35}E9$	56–70	12
$30C_5E2$	81–107	10	$35C_{35}E9$	81–123	12
$30C_5C_5$	69–99	11	$35C_{35}E11$	56–69	12
$30C_5C_5$	101–110	11	$35C_{35}E11$	117–123	11
$30C_{10}C_22$	56–72	10	$35C_{35}E13$	57–59	12
$30C_{10}C_22$	112–123	10	$35C_{35}C_51$	64–78	12
$30C_{15}C_31$	114–121	10	$35C_{35}C_52$	64–116	11
$30C_{15}C_32$	59–64	10	$35C_{35}C_53$	64–98	12

Table 3 Numbers of independent cross-link paths in 25 satellite regular networks

Group's symbol	Inclination range, deg $h = 4000$ km	$M^{\max}$	Inclination range, deg $h = 3000$ km	$M^{\max}$	Inclination range, deg $h = 2500$ km	$M^{\max}$
$25C_{25}E1$	45–130	14	51–123	11	53–58	10
$25C_5E2$	45–135	13	49–131	12	53–58	10
$25C_5E2$	—	—	—	—	105–126	10
$25C_5C_5$	58–123	13	66–114	11	—	—
$25C_{25}E2$	51–61	13	—	—	—	—
$25C_{25}E3$	47–68	12	53–56	11	—	—
$25C_{25}E4$	46–98	14	51–71	11	—	—
$25C_{25}E6$	44–133	14	49–127	11	53–98	9
$25C_{25}E6$	—	—	—	—	107–123	10
$25C_{25}E7$	44–135	14	95–128	11	103–125	10
$25C_{25}E8$	44–46	12	—	—	—	—
$25C_{25}E8$	113–136	14	123–129	10	—	—
$25C_{25}E9$	44–136	14	50–79	12	103–127	10
$25C_{25}E9$	—	—	97–131	11	103–127	10
$25C_{25}E11$	92–136	14	116–131	11	—	—
$25C_{25}E12$	51–71	13	—	—	—	—
$25C_{25}E12$	116–136	13	125–131	11	—	—
$25C_{25}C_51$	56–95	13	66–82	11	—	—
$25C_{25}C_51$	98–123	13	111–114	11	—	—
$25C_{25}C_52$	56–123	13	74–114	12	100–107	10

Results of the second research are presented in Table 3. They show how the number of paths change with the orbit height increment. Some constellations do not provide continuous coverage at the height represented in the column caption, whereas for others there are two ranges of inclination where coverage condition is fulfilled at minimum height and one range at maximum height. For better representations of the table, dashes were put in columns where such abnormalities were encountered.

To evaluate the performance of the networks for reliable information transfer purpose, the following estimation of independent informational ways might be proposed.

Consider a satellite having M satellites of the network in sight (thus a number of independent cross-link paths in the network never exceeds M). (Constructing the estimation we consider satellites to be points on unity sphere surface because only angular relationships between satellites are relevant.) It means that $M + 1$ satellites are disposed within angular distance $\varphi_{\max}$ from one (let us call it first for certainty) satellite (Fig. 5). In accordance with the continuous coverage term, the network's coverage zones (small circles of radius α) should cover the area without gaps. Thus, at least one satellite should be disposed within α limit from the point opposite to the first satellite (coverage zone of this second satellite is shown in Fig. 5). The second satellite should as well have M satellites in sight to satisfy M-path requirements. It means that at least $M + 1$ satellites are to be disposed inside a cap of angular radius $\varphi_{\max} + \alpha$ centered in the point opposite to the first satellite.

Thus, inside the caps there must be $2M + 2$ satellites out of N. The caps outline a belt on the Earth's surface. The belt should be covered without gaps in visibility zones. To estimate the number of satellites needed to cover the belt, we divide sphere surface area covered by the belt into area covered by the visibility zone and take the integer of the result. The number obtained can be positive or negative, depending on the dimensions of the caps (Fig. 6). Three cases should be considered.

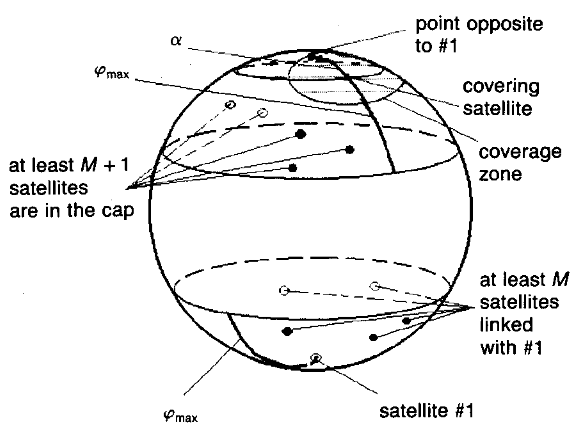
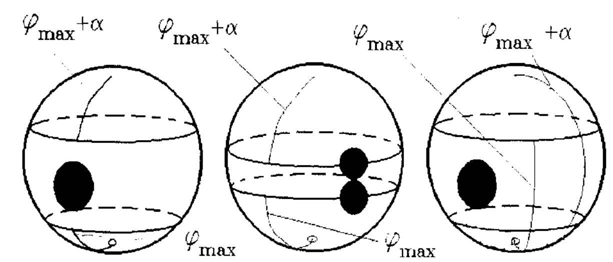


Fig. 5 Estimation of minimal number of independent cross-link paths.

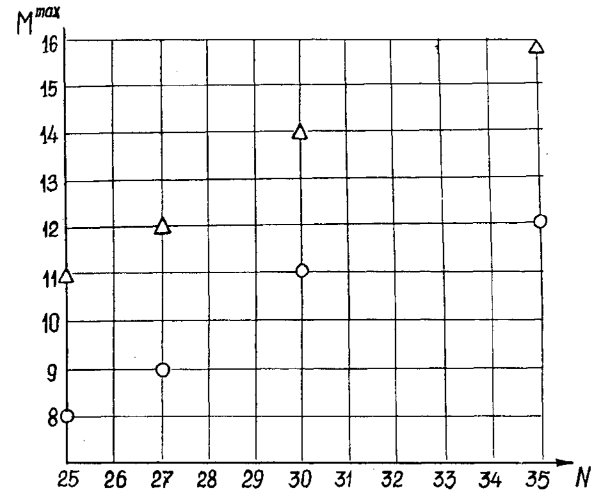


Case I. Additional satellites are needed to cover the belt

Case II. Belt can be covered by satellites from the caps

Case III. The belt is covered by excessive satellites

Fig. 6 Disposition of $M + 1$-strong caps on the Earth's surface.



O characteristics of the best kinematically regular networks

Δ - estimated value of highly obtainable performance

Fig. 7 Computation results.

Case 1

The opposite caps comprising $M + 1$ satellite each are separated with a belt that is wider than 2α, i.e.,

$$\pi - \alpha - 2\varphi_{\max} > 2\alpha$$

In this case, the belt cannot be covered without gaps by coverage zones of satellites standing at the bounds of the caps and, thus, additional satellites are needed to cover the belt's area continuously. The number M of independent paths should satisfy inequality

$$M < \text{entier}\,\frac{N - 2 - \text{entier}\,\dfrac{\Sigma_1 + \Sigma_2 - 4\pi}{2\pi(1 - \cos\alpha)}}{2} \tag{7}$$

Case 2

The belt between opposite caps or the belt of their overlapping is narrower than 2α, i.e.,

$$-2\alpha \le \pi - \alpha - 2\varphi_{\max} \le 2\alpha$$

In this case, the belt area can be covered without gaps and at the same time economically (unnecessary overlapping is minimal) with visibility zones of satellites in the caps. Thus, the estimation takes form

$$\text{M} < \text{entier}\,\frac{N - 2}{2} \tag{8}$$

Case 3

Opposite caps overlap and the belt width exceeds 2α, i.e.,

$$\pi - \alpha - 2\varphi_{\max} < -2\alpha$$

The Earth's surface can be covered continuously even if some satellites do not take part in the coverage process (the total area inside their coverage zones is no less than the area of the belt). Thus, the satellites might be excluded from coverage process and used for information transfer purposes only. The estimated value of M is given by formula (7).

In Fig. 7, the maximum number of independent cross-link paths as well as their estimated values are plotted against the number of satellites in a 2000-km-high kinematically regular network. The value of $\varphi_{\max}$ calculated for this height is $\varphi_{\max} = 78.71$ deg. Substitution of this value and value $\alpha = 35.69$ deg (after transformation into radians) in respective inequalities shows that values in Fig. 7 are obtained from formula (8).

Conclusions

The number of independent cross-link paths can be used as a measure of the information transfer reliability of a global communication network. The proposed algorithm of satellite pair independent cross-link paths computation may have significance of its own in application to problems where the number of nodally nonintersecting paths in a graph is to be computed. Kinematically regular constellations are applicable for information transfer through intersatellite cross-links: comparison of computation results with estimations showed their relatively high performance. The calculation showed that for every satellite number there exists a set of kinematically regular networks with the same value of criterion. This property of the network gives additional choice possibilities for a user.

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